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The thermal expansion of copper between 300 and 700 K

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Abstract. Measurements of the thermal expansion of copper (NBS Standard Reference Material, SRM736) have been made between 300 and 700 K. Agreement with the NBS calibration was obtained up to 630 K within 0·5 %, but at 700 K the difference amounted to 2·5 %.

1. Introduction

The copper used in this investigation was Standard Reference Material 736, supplied by the United States National Bureau of Standards. This material is supplied with a Certificate of Analysis which gives values of expansion and expansivity in the temperature range 20–800 K. Third order polynomials are also given which describe the expansivity within three temperature bands.

Measurements of the expansion of SRM 736 have been carried out at NPL between 300 and 700 K. Results are presented here for the expansion coefficient measured over several intervals within this range.

2. The material

The copper specimen, SRM 736 L3, was supplied as a rod, 6·4 mm diameter by 152 mm long. The NBS calibration certificate states that the rod was annealed at 811 K and has a purity of 99·99 at. %, with about 0·012 at. % effective (dissolved, ionised) impurities. The certificate also provides a table of expansion $\Delta L/L$ and expansivity ($\alpha = 1/L . \mathrm{d}L/\mathrm{d}T$) for temperatures at 10 K intervals between 20 and 160 K, and at 20 K intervals between 160 and 800 K. In addition, three third-order polynomials are provided which represent computed fits to the data within three temperature ranges. Details of SRM 736 and its calibration have been given by Hahn (1970).

3. The measurements

Two 25-mm lengths were prepared from the rod, with their ends flat and mutually parallel. The expansion of these two specimens was measured in an interferometric dilatometer (Bennett 1977) for several intervals within the range 300–700 K. Each specimen was placed on the base-plate of the dilatometer with a small mirror on top of it.

The expansion of the mirror (about 0·3% of the expansion of the copper) was measured separately and applied as a correction to the total measured expansion.

The temperature of the specimen was measured with a platinum/platinum–13% rhodium thermocouple, which had been manufactured from 0·3 mm wires (to BS 4937) and calibrated by the thermometry section of the Division of Quantum Metrology at NPL. This thermocouple was inserted in a radial hole drilled at the centre of the specimen, and the generated EMF was measured with a digital voltmeter (resolution 1 μV). Further thermocouples monitored the temperature distribution within the oven, and the data from all the couples were recorded at 10 min intervals by a data acquisition system.

The photoelectric signals from the interferometer detectors, in the form of phase quadrature sinusoids, were recorded on a two-pen chart recorder. The changes of length produced by each temperature increment were subsequently derived by counting the interference cycles (fringes) from the chart. The two recorded signals enable the direction of motion to be determined and reversals to be identified. Each cycle of interference corresponds to an expansion or contraction of one quarter wavelength due to the double-passed operation of the interferometer, and a temperature rise of 50 K produces an expansion of about 150 cycles of interference with the specimens used.

4. The results

The results reported in table 1 were obtained during two separate runs with the dilatometer, and all the data obtained in the two runs are included. Measurements were made over temperature intervals of approximately 80 K within the range 293–716 K. These intervals were chosen to be large enough to be measured with sufficient accuracy with a voltmeter of 1 μV resolution, while being small enough to warrant assignment of the mean expansion coefficient as the coefficient at the centre temperature of the interval.

Table 1. Measured expansion coefficients for eleven temperature intervals. Each value is the mean of six results obtained for equal time periods beginning ten minutes apart, and the fourth column gives the standard deviation of each set of six.

Mean temperature T(K)	Temperature interval ΔT (K)	Expansion coefficient α (10^{-6} K^{-1})	Standard deviation
327·1	67·3	16·96	0·04
332·5	79·7	16·85	0·02
398·4	75·3	17·52	0·03
414·8	84·0	17·80	0·02
474·2	76·4	18·06	0·02
501·8	89·8	18·32	0·05
550·2	75·6	18·61	0·02
589·7	85·7	18·84	0·03
624·8	73·7	19·11	0·05
673·8	85·2	19·55	0·03
675·2	85·5	19·76	0·03

In order to assess the random uncertainties associated with the estimation of fringe fractions from the chart and the measurement of thermocouple voltages, data were recorded for six points, 10 min apart, at the beginning and end of each interval. These were combined in pairs, so that each coefficient quoted in table 1 is the mean of six values calculated for equal time intervals beginning 10 min apart. The standard deviation of each set of six values is never greater than $0{\cdot}05\times10^{-6}$, so that the standard error of the mean does not exceed $0{\cdot}02\times10^{-6}$.

The values obtained for the expansion coefficient α are plotted in figure 1 against absolute temperature T. These values have been corrected (where necessary) for the effect of the finite temperature interval. The corrections (never greater than $0{\cdot}02\times10^{-6}$) are obtained by calculating the differences between the mean expansion coefficient $\bar{\alpha}$ and the expansion coefficient at the mean temperature α from the NBS certificate. The vertical error bars ($\pm0{\cdot}04\times10^{-6}$) indicate the total estimated uncertainty in α, as calculated from table 1 of Bennett (1977). The following third-order polynomial for α in T was fitted to the points using an unweighted orthogonal polynomial fitting procedure:

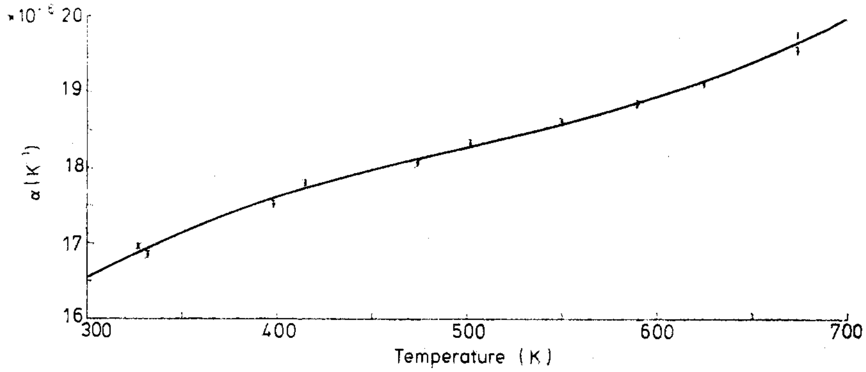

Figure 1. Measurements of the expansion coefficient of copper between 300 K and 700 K Polynomial fit for α is $\alpha\times10^{6}=6{\cdot}818+5{\cdot}639\times10^{-2}T-1{\cdot}001\times10^{-4}T^{2}+6{\cdot}626\times10^{-8}T^{3}$.

$$\alpha\times10^{6}=6{\cdot}818+5{\cdot}639\times10^{-2}T-1{\cdot}001\times10^{-4}T^{2}+6{\cdot}626\times10^{-8}T^{3}. \qquad (1)$$

The standard deviation of the fit of a polynomial of order p to n points (x_1, y_1), (x_2, y_2), $\ldots$, (x_n, y_n) is given by the expression

$$S=[(\Sigma\delta_i^{2})/(n-p-1)]^{1/2}$$

where δ_i is the residual in y of the ith point from the curve. In the case of the above third-order polynomial, S $([\Sigma\delta_i^{2}/7]^{1/2})$ is $0{\cdot}09\times10^{-6}$. This figure is rather large compared with the individual standard deviations given in table 1, suggesting that those estimates were too low. The standard deviation of a fitted value of α, calculated from equation (1), is obtained from the covariance matrix of the polynomial and does not exceed $0{\cdot}12\times10^{-6}$ in the temperature range 300–700 K.

Figure 2 illustrates the difference between the expansion coefficient calculated from equation (1) and the NBS calibration (Hahn 1970) for the temperature interval 243–775 K. The two calibrations agree within $0{\cdot}1\times10^{-6}$ (0·5 %) between 300 and 630 K, but at 700 K the coefficient calculated from equation (1) is 2·5 % greater than the NBS figure. This difference is significant, even at the 99·9 % confidence level, so that there appears to be some unresolved discrepancy between the two measurements at the high temperature end of the range.

Kollie *et al* (1973) at Oak Ridge National Laboratory, Tennessee, also obtained values for the expansion coefficient that lie above the NBS calibration for temperatures

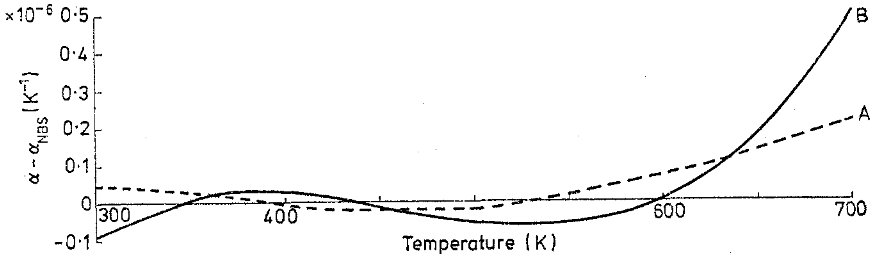

Figure 2. Differences from NBS calibration of SRM 736: A, Kollie *et al* (1973): B, this study.

above 600 K. The estimated maximum uncertainty in their measurements of α is claimed to be between 0·8% and 1·4%. They have derived the following polynomial for α from their measurements:

$$\alpha \times 10^{6} = 16{\cdot}326 - 0{\cdot}83521 \times 10^{5}\, T^{-2} + 0{\cdot}35675 \times 10^{-2}\, T + 0{\cdot}21069 \times 10^{-5}\, T^{2} \qquad (2)$$

with a residual standard deviation of fit of $0{\cdot}07 \times 10^{-6}$. The difference between this polynomial and the NBS expression is also shown in figure 2. Equation (2) does not diverge from the NBS calibration as sharply as equation (1), and the discrepancy between equations (1) and (2) is 1·5% at 700 K.

5. Conclusions

Measurements of the thermal expansion of the NBS copper reference material SRM 736 have indicated good agreement with the NBS calibration between 300 and 600 K. Above 600 K, values have been obtained which indicate that the NBS calibration may be low, as suggested by Kollie *et al* (1973). The three calibrations agree within 0·5% between 300 and 630 K and within 2·5% above 630 K. In order to resolve these remaining differences, further measurements should be undertaken to establish a generally accepted calibration for the whole temperature range, and particularly above 600 K.

Acknowledgments

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